APPLICATION NOTE: ACS 3 DIGITAL FILTERS

Application Note

ACS 3 – Digital Filters

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Objective

With this application note we would like to give you some more insight into the properties and applications of the different filter options of our new static inclinometer *ACS 3*. We hope to facilitate your choice of the proper filter option and its configurations for your specific application.

Content

Version

Version	Date	Comment
1.0	2023-04-06	First release

Chapter 1 Outlook

For our new static inclinometer platform *ACS 3*, we offer five different options to tune the angle output with respect to noise reduction:

- Moving Average Filter
- Simple Recursive Filter
- Butterworth Filter
- Critically Damped Filter
- Kalman Filter

The *Moving Average Filter* (MAF) is the most straight forward filter and offers a high degree of intuition. As the only offered option with a finite impulse response, it provides a precisely defined delay. Therefore, we have chosen to use this option as our default setting.

The *Simple Recursive Filter* is also relatively plain and offers a fast reaction upon any position change. However, the recursiveness results in a strong dependency on the sensor's position history.

For the new generation of the POSITAL static inclinometer we have introduced the *Butterworth Filter* as a very classical frequency filter. The major advantage of this filter is its attenuation characteristics in the frequency domain. In other words, the filter can damp out signal noise in a specified frequency range. A downside, however, is that this filter tends to overshoot at certain settings.

The *Critically Damped Filter* has been implemented exactly to solve this disadvantage. It as well is a frequency filter, but its attenuation characteristics is not as pronounced as the Butterworth Filters'. Yet even for higher filter orders, this filter does not overshoot under any condition.

Finally, we built a rather sophisticated *Kalman filter* into our inclinometer. The idea was to provide a filter option which can suppress signal noise very strongly when the sensor is stationary as well as react dynamically to any change in tilt position.

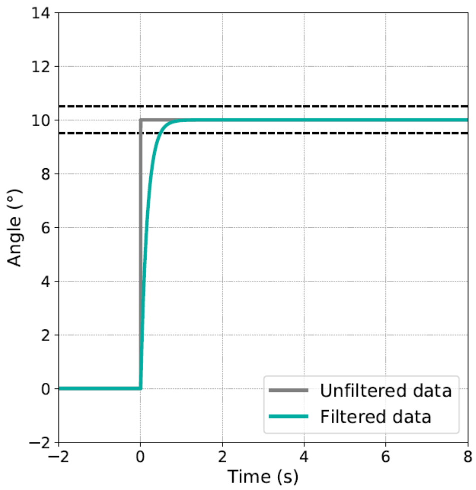
Chapter 2 Introduction to Digital Filters

Purpose & side effects:

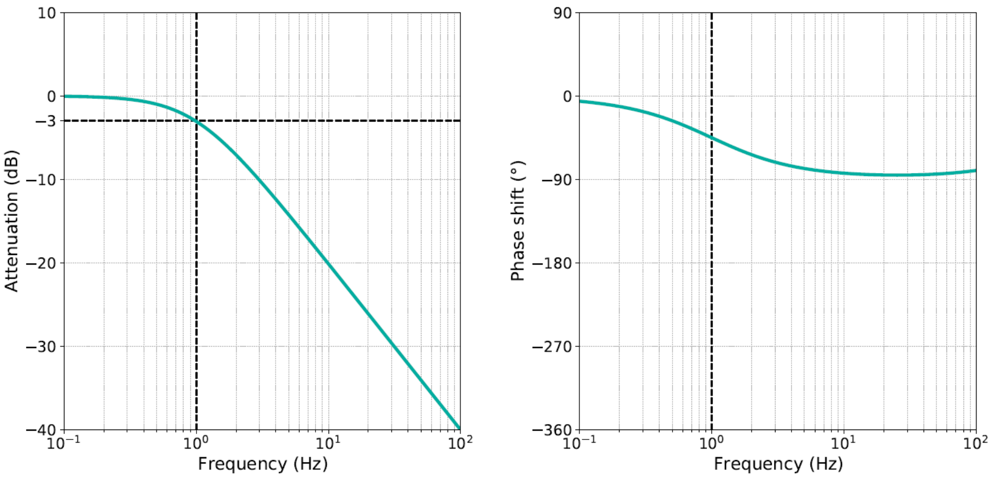
- General function of digital filters:
  Manipulation of discrete signals to reduce or enhance certain signal aspects
- Application in inclinometers: Smoothing noisy data
  Reduction in amplitude of selected signal components within a dedicated frequency range
  Low Pass Filter (LPF): Suppression of frequencies above a certain threshold
- Downside of filtering: Time lag
  Every digital low pass filter will cause a delay of the filtered signal compared to the original one

Properties:

- Time domain:
  To describe the filter's time-dependent behavior, usually its step response is analyzed.
  Here, an input data set is used, which shows a perfectly sharp "step" between two dedicated positions and the effect of the considered filter on this input data is displayed. The introduced delay, the additional time needed for the filtered data to reach and stay within a ±5% error band around the unfiltered signal, is also referred to as *settling time*.



- Frequency domain:
  For an analysis of the filter's behavior in the frequency domain, its frequency response is displayed in form of a so-called Bode plot.
  Here, sine waves of various frequencies are the input signals on which the filter is applied. The change in amplitude (attenuation) and the shift in phase compared to the input signal is plotted against the excitation frequency.
  In general, the frequency at which the amplitude experiences an attenuation down to $1/\sqrt{2}$ of the original input signal (i.e., –3 dB) is referred to as cut-off frequency $f_{cut\text{-}off}$.



- Balancing act between time and frequency domain:
  The reciprocal relation between time and frequency is reflected in the filter properties: A desirable signal behavior in the step function usually results in a not quite so sharp transition between the pass and stop band. In return, a steep transition curve in the frequency domain usually results in overshooting and ringing as well as a delay in the step response. Consequently, the "perfect" filter adjustment will always be a trade-off.

Classification:

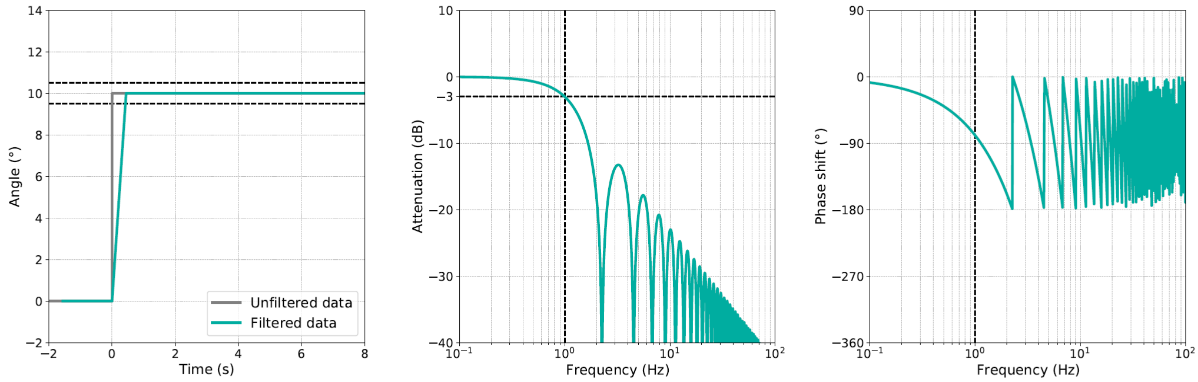
- Conventional digital filters can be classified by their impulse response:
  - Finite Impulse Response (FIR):
    Non-recursive: Uses only input signals
    Window method: Convolution of the signal with a weighting function
    Advantage: Ensured stability
    Representatives: *Moving Average Filter*
  - Infinite Impulse Response (IIR):
    Always recursive: Uses both input signals and previous output signals
    Characterized by its transfer function
    Advantage: Tailored solutions available for specific frequency domain requirements
    Representatives: *Butterworth Filter*, *Critically Damped Filter*, *Simple Recursive Filter*
- In contrast to the classical FIR and IIR filters acting by signal and time series analysis, the *Kalman filter* is based on state-space modeling. Its algorithm uses a time series of measurements, including statistical noise and other inaccuracies, and produces estimates of unknown variables that tend to be more accurate than those based on a single measurement alone, by estimating a joint probability distribution over the variables for each timeframe. In its time behavior, it is very similar to an IIR filter with a delay stage.

Below you will find a more detailed explanation of each of our filter options. A graph of the step response is shown as well as a Bode plot for the frequency-related filter properties. For the conventional filter types, an exemplary cut-off frequency of 1 Hz has been chosen.

Chapter 3 Moving Average Filter

Application:

- Default filter mode setting for ACS 3
- High degree of stability and predictability
- Focus on step response, not on frequency domain



Details:

- Special case of FIR filter:
  Usage of an equal distribution as weighting function in the convolution
- Step response:
  Quickest possible reaction (optimal delay)
- Frequency response:
  Harsh ripples in stop band

Parameter	Unit	Minimum value	Maximum value	Default value
Length	ms	1	5'000	200

The depicted cut-off frequency of 1 Hz is reached using a MAF length of 443 ms.

Increasing the MAF length results in an increased settling time. The steepness of the slope in the step response decreases linearly with it, while the attenuation and phase shift curves of the Bode plot are shifted towards lower frequencies.

Remark
Depending on the communication interface of the *ACS 3* inclinometer, more than one filter parameter might be available. For more details, please refer to the corresponding product manual.

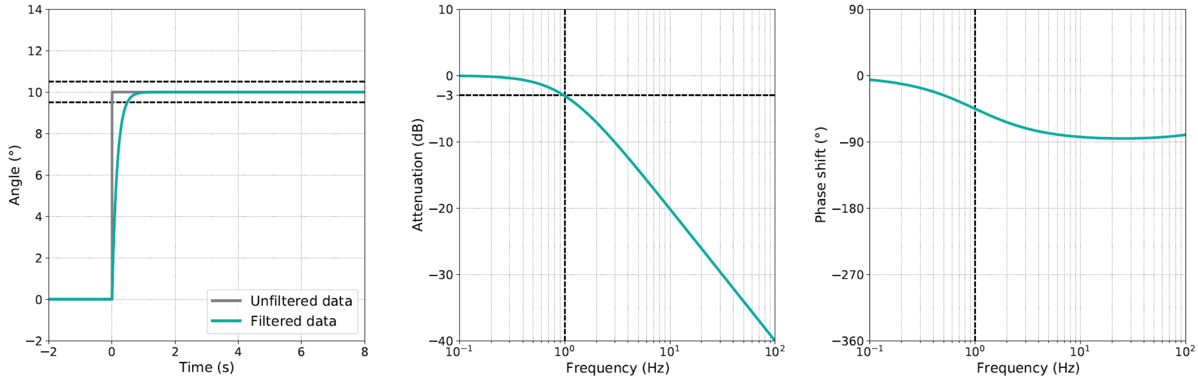
Chapter 4 Simple Recursive Filter

Application:

- Focus on step response:
  No overshoot and ringing

Properties:

- Simplest form of IIR filter
- Weighting of new unfiltered data with last filtered value:
  $y_n = \mathrm{a} \bullet x_n + (1 - a) \bullet y_{n-1}$
  Legend: n: filter iteration | a: filter coefficient | x: signal input | y: filter output



- Step response:
  Quick response time
- Frequency response:
  Shallow transition between pass- stopband

Parameter	Unit	Minimum value	Maximum value	Default value
Coefficient	‰	1	999	990

The depicted cut-off frequency of 1 Hz is reached using a coefficient of 997‰.

Increasing the coefficient results in an increased settling time. The exponential function in the step response is stretched, while the attenuation and phase shift curves of the Bode plot are shifted towards lower frequencies.

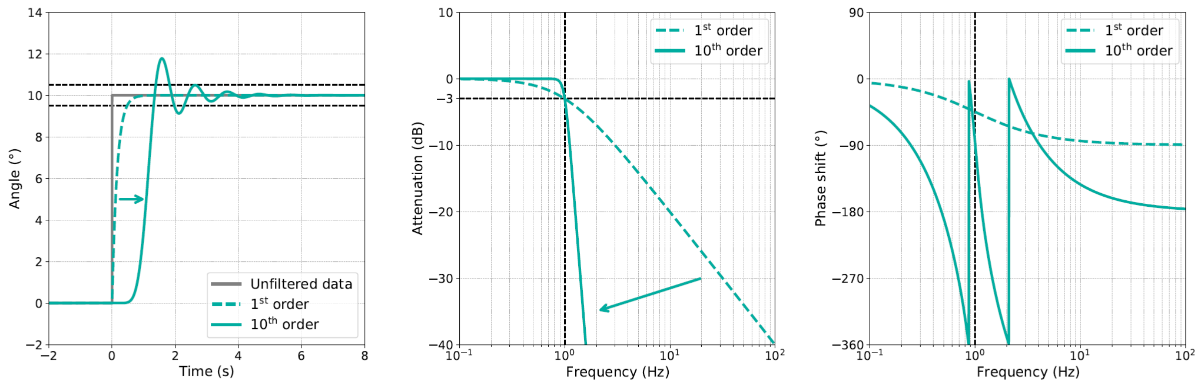
Chapter 5 Butterworth Filter

Application:

- Focus on frequency response:
  Suppression of frequencies above a certain threshold
- High flexibility

Properties:

- Most simple type of classic frequency filters
- No ripples in pass- and stopband



- Step response:
  Delay, overshooting and ringing increasing with filter order
- Frequency response:
  Maximally flat in the passband (no ripples at all)
  Edge steepness increasing with filter order

Parameter	Unit	Minimum value	Maximum value	Default value
Order	-	1	10	1
Cut-off frequency	mHz	1	65’535	1’000

The order of the Butterworth filter has a significant impact on both step response and Bode plot. In the step response, the 1st filter order does not lead to any overshoot of the filtered signal, while for each additional order step the number of oscillations is increased. At the same time, the time lag is growing. In the frequency response the steepness of the transition between passband and stopband is increased by –20dB per order of magnitude in frequency with each additional filter order.

An increase in cut-off frequency will lead to a compression of the step response and a shift towards higher frequencies in the Bode plot.

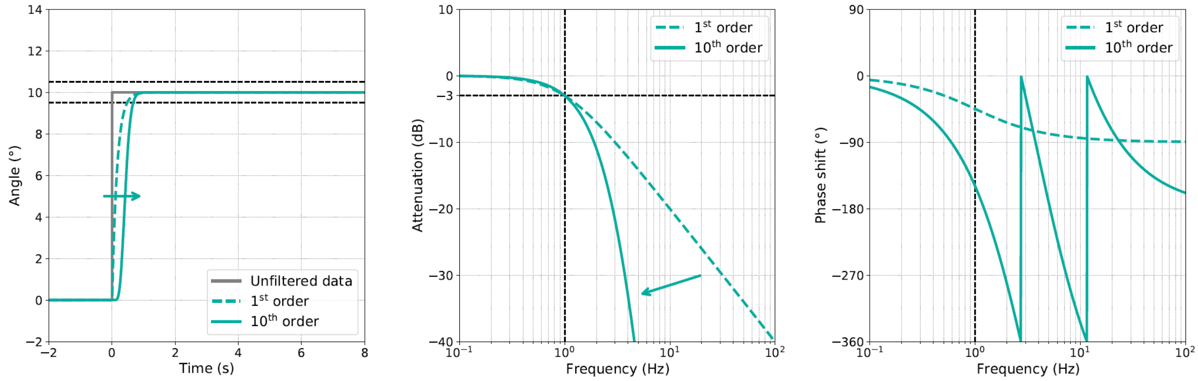
Chapter 6 Critically Damped Filter

Application:

- Compromise for focus on step AND frequency response:
  No overshooting and ringing
  Suppression of frequencies above a certain threshold with flatter transition
- High flexibility

Properties:

- Special case of IIR: Cascade of 1st order frequency filters
  No overshoot or ringing possible
  Lowered edge steepness compared to Butterworth filter



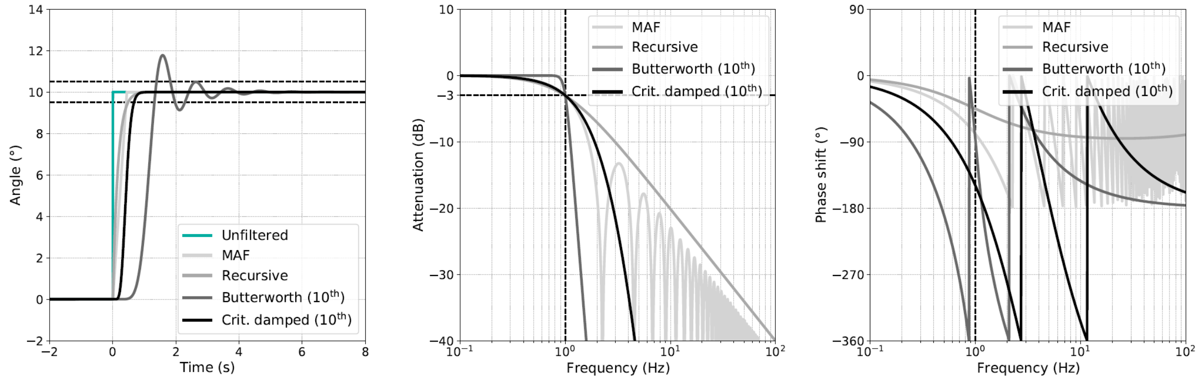
- Step response:
  No overshooting and ringing
  Delay increasing with filter order
- Frequency response:
  Edge steepness increasing with filter order

Parameter	Unit	Minimum value	Maximum value	Default value
Order	-	1	10	1
Cut-off frequency	mHz	1	65'535	1'000

The order of the critically damped filter has a distinct, but significantly lower impact on both step response and Bode plot compared to the Butterworth filter. In the step response none of the orders will lead to an overshoot, still the time lag is increasing with the increasing order. In the frequency response the steepness of the transition between passband and stopband is increased by each additional order, but far not as rapidly as for the Butterworth filter.
An increase in cut-off frequency will lead to a compression of the step response and a shift towards higher frequencies in the Bode plot.

Chapter 7 Comparison of conventional filters

When comparing the previously discussed conventional digital filters, massive differences in their time and frequency domain are visible.



The graphs above show a comparison between the *Moving Average Filter (MAF)*, a *Simple Recursive Filter* and both a 10th order *Butterworth* and *Critically Damped Filter*, all having a cut-off frequency of 1 Hz.

The *MAF* has the best step response in form of a linear position transition within a deterministic time delay without any overshoot, while in the frequency domain it shows harsh ripples in the stop band upon a rather gradual transition range.
This filter type should be used whenever a plain filter behavior is required and the focus is set on the time domain. It is recommended for the removal of strong white noise and set as our factory default due to its high predictability.

A significantly different behavior can be seen for the *Butterworth Filter*. Here, we see an overshooting and ringing behavior on the step response in the time domain, while in the frequency domain the transition between passband and stopband is very steep without any kind of ripples in the amplitude and phase shift.
This type of filter is intended to block interfering frequencies quite specifically. If the sensor is experiencing a disturbing frequency well below the time horizon of the dynamics of the motion profiles of interest, this filter can help to block out the disturbances while rather lightly affecting the actual angle signal.

As a compromise, we have introduced the *Critically Damped Filter*, which combines the positive properties of the two filters mentioned above, but in a somewhat attenuated form. It has a relative steep step response without the risk of overshooting and ringing, while showing a relatively steep transition in the frequency domain without forming any ripples.
It can be used whenever a disturbing frequency has to be banned from the actual signal without sacrificing the step response quality in the time domain.

The *Simple Recursive Filter* has a very comparable behavior to the frequency filters (Butterworth Filter & Critically Damped Filter) in the special case of using their first filter order. It has been kept in our portfolio for backwards-compatibility reasons.

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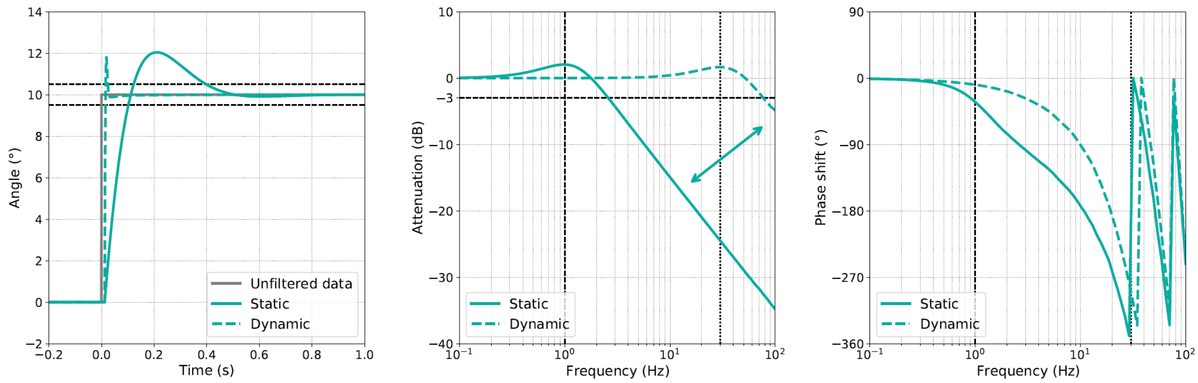
Chapter 8 Kalman Filter

Application:

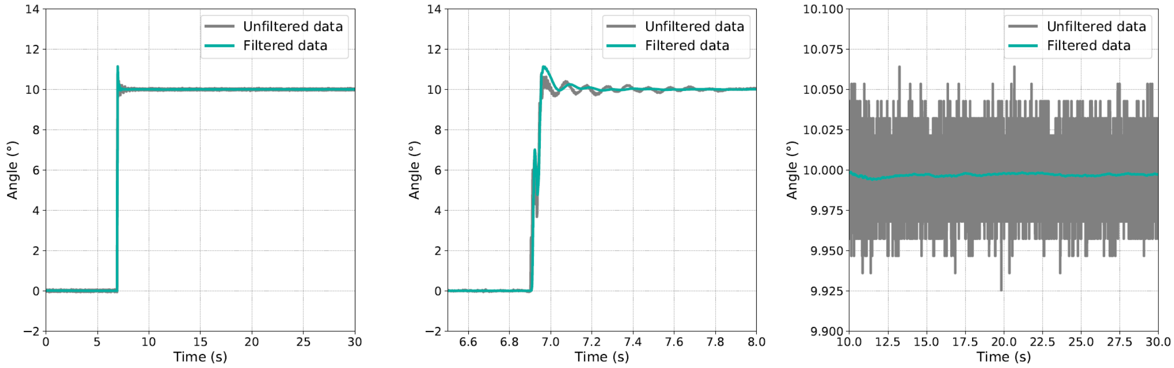
- Dynamic environment
- Focus on time domain:
  Combines highly dynamic step response (dynamic mode) with strong noise smoothing (static mode)
- Attention:
  - A dedicated knowledge of the system is required to achieve a desired filter performance
  - The statistical nature of the filter and its adaptivity cause changes in time delay which might be unwanted when used as input for control units

Properties:

- (Optionally) adaptive:
  Switches automatically between a static and dynamic mode
- State-space type filter (also recursive, IIR)
- Estimation filter based on statistical processes



- Following the presentation of the time and frequency characteristics of the conventional filters, two different example curves are depicted, which – this time – do not represent two different filter settings, rather their combination is intended to reflect the behavior of the Kalman filter as a direct consequence of its adaptivity.
- Step response:
  Overshooting and slight ringing (static mode NOT used upon position step)
- Frequency response:
  Small enhancement at resonance frequency
- Adaptivity:
  Static mode is supposed to filter strongly, while the dynamic mode has a rather low filtering effect. The transition between the two modes depends on the measured sensor data in combination with a predefined threshold value.



The graphs above show an exemplary trace of measured data including signal noise upon a steep 10° step as well as the filtered signal using an (adaptive) *Kalman filter*. While the left graph shows the complete curves, the middle and the right graphs depict zoom-ins into specific time and angle ranges to illuminate special filter characteristics.
The settings have been chosen to fulfil the following criteria: The filtered data should follow the step function as rapidly as possible (middle graph), while the signal noise should be damped out massively upon stillstand (right). The shown signal has been filtered using default settings.

Parameter	**Unit**	**Minimum value**	**Maximum value**	**Default value**
Static coefficient	%	0	100	70
Motion threshold	*mg*	0	1'000	2
Dynamic coefficient	%	0	100	50

The motion threshold has to be carefully adjusted to the system noise. The chosen value represents an internal limit, which is used to switch between two – occasionally diametral – filter characteristics. As a guideline, you could record the accelerometer raw data of the inclinometer in a machine state, where the machine is switched on, but not moving at all. As analysis of the recorded trace, you need to identify the peak-to-peak noise and choose the motion threshold well above it. Ten times the standard deviation is typically a good starting point.

The static coefficient will define the filter strength whenever a static situation is detected. Here, you can control the filter strength within a range of 0 to 100%, while 0% corresponds to no smoothening of the input data and 100% to a very strong filter behavior, comparable to a very small cut-off frequency for conventional filters.

The dynamic coefficient will define the filter strength for a dynamic scenario in dependence on the static setting. Again, the filter strength can be controlled within a range between 0% and 100%. 0% corresponds to no deviation from the filter behavior defined for a static situation, although a dynamic situation is detected. Increasing the adjustment screw towards its maximum will result in practically unfiltered values during a dynamic event.

Chapter 9 Conclusion

The choice of a digital filter is very decisive for the data quality and needs to be adjusted custom-fit to each individual application.

For our new generation of static inclinometers, we offer four different conventional filter options, which are deterministic and differ strongly in their behavior both in time and frequency domain. Beyond, we offer a *Kalman filter* which has the big advantage of optional adaptivity. This filter is not recommended to be used with automated controls but can lead to convincingly good results when set-up correctly and used for manual input.

When choosing a filter option beyond our default sensor configuration, it is recommended to first collect the requirements list and select the filter type accordingly. In the next step, the filter specific configurations can be adjusted.

The following table is supposed to give you a first idea of how a filter type could be chosen. However, it cannot cover all use cases and is only intended as a first indication. Moreover, only the general filter characteristics are considered, while the individual filter specific configurations are neglected.

Filter type	**Filter order**	**Static smoothening**	**Settling time**	**No overshoot**	**Frequency suppression**	**Convergence**	**Robustness**	**Configuration complexity**
Moving Average Filter		+	+	+ +	–	+ +	+ +	+ +
Recursive Filter		+	o	+ +	o	+	+	+
Butterworth Filter	1 ⋮ 10	+ ⋮ + +	o ⋮ –	+ + ⋮ – –	o ⋮ + +	+ ⋮ –	+ ⋮ o	o ⋮ o
Critically Damped Filter	1 ⋮ 10	+ ⋮ + +	o ⋮ –	+ + ⋮ + +	o ⋮ +	+ ⋮ –	+ ⋮ +	o ⋮ o
Kalman Filter		+ +	+ +	–	– –	o	–	– –

Definition of used terms:

Static smoothening	Potential damping of signal noise in a static environment (++ strong)
Settling time	Time the filtered signal needs to reach and stay within a certain error band around the final position value after a step-shaped motion (++ very short)
No overshoot	Inverted ability of the filter to overshoot (++ no overshoot possible)
Frequency suppression	Sharpness of transition to band of suppressed frequencies (++ very steep)
Convergence	Filter ability to not depend on history beyond the settling time (++ finite foretime impact)
Robustness	Filter ability to resist resonance excitations (++ very robust)
Configuration complexity	User know-how required to successfully configure the filter (++ easy)